Optimization of polling systems by means of gradient methods and the power-series algorithm

Blanc, J.P.C.; van der Mei, R.D.

Publication date:
1992

TILBURG UNIVERSITY
KATHOLIEKE UNIVERSITEIT BRABANT
POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS

[redacted]

Queuing Theory

DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

OPTIMIZATION OF POLLING SYSTEMS BY MEANS OF GRADIENT METHODS AND THE POWER-SERIES ALGORITHM

J.P.C. Blanc, R.D. van der Mei

FEW 575

Communicated by Prof.dr.ir. O.J. Boxma

OPTIMIZATION OF POLLING SYSTEMS BY MEANS OF GRADIENT METHODS AND THE POWER-SERIES ALGORITHM

J.P.C. Blanc
R.D. van der Mei
Tilburg University, Faculty of Economics
P.O. Box 90153, 5000 LE Tilburg, The Netherlands

Abstract

This paper is devoted to numerical optimization of polling systems with a general service order table and with Bernoulli service disciplines at each visit of the server to a queue. The optimization is based on the use of the power-series algorithm (p.s.a.), a tool which can be applied for the numerical evaluation of performance measures for these models. The applicability of the p.s.a. is extended towards the numerical computation of first order partial derivatives with respect to the Bernoulli parameters. This extension strongly improves the efficiency of procedures for optimization purposes.

Keywords: polling systems, optimization, Bernoulli schedules, power-series algorithm, derivatives

1. Introduction

Polling systems are queueing models in which several classes of customers are served by a single server. Such systems are widely used to model computer and communication systems. The reader is referred to [14] for an overview of the available solution techniques and to [11] for an overview of the range of applications.
In the present paper, the order in which the server visits the queues is prescribed by a general service order table of finite length, a so-called polling table. At each queue the customers are served according to a Bernoulli service discipline, as introduced in [9]. A Bernoulli service discipline stochastically limits the number of customers that is served during one visit of the server to a particular queue. For a given polling table the Bernoulli parameters serve as decision variables; a vector of Bernoulli parameters is referred to as a Bernoulli schedule. The present paper is devoted to optimization of Bernoulli schedules; the problem is to find for a given polling table a Bernoulli schedule which minimizes an arbitrary weighted sum of the steady-state mean waiting times at the various queues.

For polling systems with Bernoulli schedules, detailed analysis is only possible in special cases (cf. [9, 15, 16]). As a consequence, there is a need for numerical algorithms for the evaluation of performance measures of the system. In this paper we focus on the use of the p.s.a., introduced in [8] and developed in [1-4]. The p.s.a. provides a means to evaluate performance measures of multi-queue systems which can be modeled as multi-dimensional quasi birth-death processes. Polling systems form an important class of systems which can be modeled by such a process, cf. [4] for a discussion on the implementation of the present model into the p.s.a.. Leung [10] has proposed another numerical technique based on discrete Fourier Transforms to compute queue length distributions in polling models with a so-called probabilistically-limited service strategy; the Bernoulli service strategy is a special case of such a service strategy. The main disadvantage of both algorithms is that the time and memory requirements increase exponentially with the number of queues, so that their use is restricted to systems with a moderate number of queues.

Optimization of Bernoulli schedules for *cyclic* polling systems has been discussed in Blanc and van der Mei [5], who propose to use the p.s.a. combined with the conjugate gradient method (cf. [13]). The disadvantage of the approach in [5] is the fact the partial derivatives with respect to the Bernoulli parameters, which are required for this method, are estimated based on finite differences. These finite differences (and the step sizes in the line search) have to be adapted to the accuracy of the computed value of the performance measure. However, the latter can only be estimated on the basis of the last few computed terms of the

power-series. Moreover, in order to estimate partial derivatives with respect to the Bernoulli parameters, the performance of neighbouring Bernoulli schedules has to be evaluated, which may be a rather time consuming task.
To overcome these difficulties, for the present model the use of the p.s.a. is extended towards the numerical computation of partial derivatives of the performance measures with respect to the Bernoulli parameters up to, in principle, any level of accuracy. This extension leads to a considerable reduction of the time requirements of the optimization procedure in [5]. Moreover, it allows the analysis of the sensitivity of the performance measures with respect to the Bernoulli parameters.
Summarizing, the contribution of this paper is twofold: (i) a method is developed to calculate partial derivatives of the performance measures with respect to the Bernoulli parameters; (ii) the results of [5] are extended to polling systems with a general service order table. We emphasize that the basic idea of the approach discussed in the present paper can naturally be extended towards a more general type of parameters within the broad class of queueing systems for which the p.s.a. is applicable, cf. [4] (Section 1) for a variety of systems which are contained in this class. For instance, the same approach can be applied to other routing mechanisms such as state-dependent routing (e.g., priority for the longest queue) and random polling. The approach also allows for optimization of, e.g., routing probabilities in systems with random polling and with given service disciplines at all queues.

The rest of the paper is organized as follows. In Section 2 a detailed description of the model is given. In Section 3 we deduce differential equations for the partial derivatives of the state-probabilities with respect to the Bernoulli parameters. In Section 4 we derive a recursive scheme to compute simultaneously the state-probabilities and their partial derivatives with respect to the Bernoulli parameters. In Section 5 efficient use of the computation time and available memory space for the implementation of the p.s.a. for the present model is discussed. In Section 6 we compare the performance of the optimization procedure based on the approach presented in this paper, in which partial derivatives are calculated by means of the p.s.a. with the approach proposed in [5], in which partial derivatives are estimated based on finite differences.

2. Model description and problem formulation

Consider a continuous-time system consisting of s queues, which are served by a single server. Customers arrive at queue i according to a Poisson process with rate λ_i , $i=1, \dots, s$. Each queue may contain an unbounded number of customers. The order in which the server inspects the queues is determined by a so-called polling table, described by a surjective mapping

$$l : \{1, \ldots, L\} \to \{1, \ldots, s\},$$

where L is the length of the polling table. The server visits the queues according to the polling table periodically, i.e., the number of the queue which is visited during the i-th visit of the server to a queue is $l(i \; mod \; L)$, $i=1, 2, \ldots$, with the convention $l(0)=l(L)$. The number of customers served during one visit of the server to a particular queue is determined by a Bernoulli schedule, i.e., by a vector $\mathbf{q}=(q_1, \ldots, q_L)$ of probabilities which are used as follows. When the server arrives at a non-empty queue, at least one customer at that queue is served; otherwise, the server proceeds to the next queue according to the polling table. After a service completion at queue $l(h)$, $h=1, \ldots, L$, the server starts serving another customer at queue $l(h)$ with probability q_h, provided this queue has not yet been emptied; otherwise, the server proceeds to queue $l(h+1)$ according to the polling table. At each queue the customers are served on a first-in-first-out basis.

The service times of customers arriving at queue i are assumed to be distributed according to a Coxian distribution with k-th moment $\beta_i^{(k)}$, $k=1, 2, \ldots$, $i=1, \ldots, s$. The Coxian service time distribution at queue i, $i=1, \ldots, s$, consists of Ψ_i^1 phases; with probability $\pi_i^{1,\varphi}$, a service is composed of phases $\varphi, \varphi-1, \ldots, 1$, $\varphi=1, \ldots, \Psi_i^1$, and the transition rate from phase ψ is $\mu_i^{1,\psi}$, $\psi=1, \ldots, \Psi_i^1$. Consequently, the Laplace-Stieltjes Transform (L.S.T.) of the service time distribution at queue i, $i=1, \ldots, s$, is given by

$$\beta_i(\omega)=\sum_{\varphi=1}^{\Psi_i^1} \pi_i^{1,\varphi} \prod_{\psi=1}^{\varphi} \frac{\mu_i^{1,\psi}}{\mu_i^{1,\psi}+\omega}, \quad Re\ \omega \geq 0. \tag{1}$$

The times which are needed to switch from queue $l(h-1)$ to queue $l(h)$ are also assumed to be distributed according to a Coxian distribution, with k-th moment $\sigma_h^{(k)}$, $k=1, 2, \ldots$, and with parameters Ψ_h^0, $\mu_h^{0,\varphi}$, $\pi_h^{0,\varphi}$, $\varphi=1, \ldots, \Psi_h^0$, which are defined in a similar way as those of the service time distributions, $h=1, \ldots, L$.

All service times and switch-over times are assumed to be mutually independent and independent of the state of the system.

The sum of the arrival processes at the various queues is a Poisson process with rate

$$\Lambda:=\sum_{i=1}^{s} \lambda_i . \tag{2}$$

The L.S.T. of the service time of an arbitrary customer is $\beta_i(\omega)$ with probability λ_i/Λ, $i=1, \ldots, s$, and hence the k-th moment β_k of the service time distribution of an arbitrary customer is given by

$$\beta_k = \sum_{i=1}^{s} \frac{\lambda_i}{\Lambda} \beta_i^{(k)}, \quad k=1,2,\ldots . \tag{3}$$

The offered load ρ_i to station i and the total offered load ρ to the system are defined by

$$\rho_i := \lambda_i \beta_i^{(1)}, \quad i=1,\ldots,s, \quad \rho := \sum_{i=1}^{s} \rho_i . \tag{4}$$

Because ρ will be used as a variable in the p.s.a., we define the following quantity

$$a_i := \lambda_i / \rho , \tag{5}$$

referred to as the relative arrival rate at queue i, $i=1, \ldots, s$.
Denote the k-th moment of the total switch-over time during one cycle of the server along the queues by σ_k, $k=1, 2, \ldots$.
Necessary and sufficient conditions for the stability of polling systems have been derived in [7]. For the present model with a general polling table and a Bernoulli schedule these conditions read:

$$\rho \, (1 + \sigma_1 \max_{i=1,\ldots,s} \{ a_i / M_i \}) < 1, \tag{6}$$

where M_i is the mean of the maximal number of customers that may be served when the server goes along the queues in the order listed on the polling table, i.e.,

$$M_i := \sum_{h=1}^{L} \frac{I\{ l(h) = i \}}{1 - q_h}, \quad i=1,\ldots,s; \tag{7}$$

here, $M_i = \infty$ when there is at least one h, $h=1, \ldots, L$, with $l(h)=i$ and $q_h=1$; $I\{E\}$ stands for the indicator function on event E. Denote by ρ_0 the value of ρ for which equality holds in (6).
In the sequel it is assumed that the stability condition (6) is satisfied and that the system is in steady-state.

Optimization problem
The optimization problem is to minimize the cost function, defined as

$$C(\boldsymbol{q}):=\sum_{i=1}^{s} c_i EW_i, \tag{8}$$

over all Bernoulli schedules for which (6) holds; here c_i , $i=1, \ldots, s$, are strictly positive cost coefficients and EW_i , $i=1, \ldots, s$, are the steady-state mean waiting times at the various queues. The optimal Bernoulli schedule will be denoted by $\mathbf{q}^*$.

In general, the optimization problem is not analytically solvable, but may be solved numerically by using the p.s.a. combined with some optimization procedure. In Sections 3 and 4 we extend the use of the p.s.a. towards the computation of partial derivatives of the state-probabilities with respect to the Bernoulli parameter. This extension leads to a considerable improvement of the efficiency of the optimization procedure.

3. Balance equations; derivatives

In this section we extend the p.s.a. applied to the present model (cf. [4]) towards the computation of derivatives with respect to the Bernoulli parameters. In order to apply the p.s.a. we first define a continuous time Markov process to describe the system. Then, we deduce partial differential equations for the derivatives of the state-probabilities with respect to the Bernoulli parameters. These equations follow readily from the balance equations for the state-probabilities.

For the present model let $\mathbf{N}=(N_1, \ldots, N_s)\in\mathbf{N}^s$ denote the vector of numbers of customers at each of the queues. In order to transform the queue length process into a Markov process, it is most appropriate to introduce a triple (H, Z, Φ) of supplementary variables. The variable H will indicate the actual position on the polling table (the value of H, in the range $1, \ldots, L$, changes at instants at which the server leaves a queue); the variable Z will indicate whether the server is switching (Z=0) or serving (Z=1); the variable Φ will indicate the actual phase of either the switch-over time or the service time. In general, the supplementary state space is not the same for all $\mathbf{N}=\mathbf{n}$. However, for simplicity of the discussion it will be assumed throughout that the supplementary state space is the same for all $\mathbf{n}$, $\mathbf{n}\in\mathbf{N}^s$, and is given by

$$\Theta := \{1,\ldots,L\} \times \{0,1\} \times \{1,\ldots,\max_{i,j} \Psi_i^j\}, \quad (i,j)\in\{1,\ldots,L\}\times\{0,1\}, \tag{9}$$

while it is possible that some states cannot be entered.

The state-probabilities are defined as follows: for $\mathbf{n}\in\mathbf{N}^s$, $h=1,\ldots,L$, $\zeta=0,1$, $\varphi=1,\ldots,\Psi_h^0$ if $\zeta=0$, $\varphi=1,\ldots,\Psi_{l(h)}^1$ if $\zeta=1$,

$$p(\mathbf{n},h,\zeta,\varphi) := Pr\{(N,H,Z,\Phi)=(\mathbf{n},h,\zeta,\varphi)\}. \tag{10}$$

The balance equations read (cf. [4]), for $\mathbf{n}\in\mathbf{N}^s$, $h=1,\ldots,L$, $\varphi=1,\ldots,\Psi_h^0$,

$$\begin{aligned}\left[\rho\sum_{i=1}^{s}a_i+\mu_h^{0,\varphi}\right]p(\mathbf{n},h,0,\varphi)&=\rho\sum_{i=1}^{s}a_i p(\mathbf{n}-\mathbf{e}_i,h,0,\varphi)I\{n_i>0\}\\&+\mu_h^{0,\varphi+1}p(\mathbf{n},h,0,\varphi+1)I\{\varphi<\Psi_h^0\}+\mu_{h-1}^{0,1}\pi_h^{0,\varphi}p(\mathbf{n},h-1,0,1)I\{n_{l(h-1)}=0\}\\&+\mu_{l(h-1)}^{1,1}\pi_h^{0,\varphi}p(\mathbf{n}+\mathbf{e}_{l(h-1)},h-1,1,1)\left[1-q_{h-1}I\{n_{l(h-1)}>0\}\right];\end{aligned}\tag{11}$$

and for $\mathbf{n}\in\mathbf{N}^s$, $h=1,\ldots,L$, $\varphi=1,\ldots,\Psi_{l(h)}^1$, $n_{l(h)}>0$,

$$\begin{aligned}\left[\rho\sum_{i=1}^{s}a_i+\mu_{l(h)}^{1,\varphi}\right]p(\mathbf{n},h,1,\varphi)&=\rho\sum_{i=1}^{s}a_i p(\mathbf{n}-\mathbf{e}_i,h,1,\varphi)I\{n_i>0\}\\&+\mu_{l(h)}^{1,\varphi+1}p(\mathbf{n},h,1,\varphi+1)I\{\varphi<\Psi_{l(h)}^1\}+\mu_h^{0,1}\pi_{l(h)}^{1,\varphi}p(\mathbf{n},h,0,1)\\&+q_h\mu_{l(h)}^{1,1}\pi_{l(h)}^{1,\varphi}p(\mathbf{n}+\mathbf{e}_{l(h)},h,1,1).\end{aligned}\tag{12}$$

Because the server cannot be serving at a queue which is empty, we have for $h=1,\ldots,L$, $\varphi=1,\ldots,\Psi_{l(h)}^1$,

$$p(\mathbf{n},h,1,\varphi)=0,\quad \text{if}\quad n_{l(h)}=0. \tag{13}$$

Further, according to the law of the total probability,

$$\sum_{(\mathbf{n},h,\zeta,\varphi)\in\mathbf{N}^s\times\Theta}p(\mathbf{n},h,\zeta,\varphi)=1. \tag{14}$$

We introduce partial derivatives of the state-probabilities with respect to q_r, $r=1,\ldots,L$, for $\mathbf{n}\in\mathbf{N}^s$, $h=1,\ldots,L$, $\zeta=0,1$, $\varphi=1,\ldots,\Psi_h^0$ if $\zeta=0$, $\varphi=1,\ldots,\Psi_{l(h)}^1$ if $\zeta=1$,

$$p_r(\mathbf{n},h,\zeta,\varphi) := \frac{\partial}{\partial q_r}p(\mathbf{n},h,\zeta,\varphi). \tag{15}$$

The equations for these derivatives follow from (11) and (12) by differentiating both sides with respect to q_r, $r=1,\ldots,L$. For $\mathbf{n}\in\mathbf{N}^s$, $h=1,\ldots,L$, $\varphi=1,\ldots,\Psi_h^0$,

$$\left[\rho\sum_{i=1}^{s}a_i+\mu_h^{0,\varphi}\right]p_r(\boldsymbol{n},h,0,\varphi)=\rho\sum_{i=1}^{s}a_i p_r(\boldsymbol{n}-\boldsymbol{e}_i,h,0,\varphi)I\{n_i>0\}$$
$$+\ \mu_h^{0,\varphi+1}p_r(\boldsymbol{n},h,0,\varphi+1)I\{\varphi<\Psi_h^0\}\ +\ \mu_{h-1}^{0,1}\pi_h^{0,\varphi}p_r(\boldsymbol{n},h-1,0,1)I\{n_{l(h-1)}=0\} \tag{16}$$
$$+\ \mu_{l(h-1)}^{1,1}\pi_h^{0,\varphi}p_r(\boldsymbol{n}+\boldsymbol{e}_{l(h-1)},h-1,1,1)\left[1-q_{h-1}I\{n_{l(h-1)}>0\}\right]$$
$$-\ \mu_{l(h-1)}^{1,1}\pi_h^{0,\varphi}p(\boldsymbol{n}+\boldsymbol{e}_{l(h-1)},h-1,1,1)I\{n_{l(h-1)}>0\};$$

and for $\mathbf{n}\in\mathbf{N}^s$, $h=1,\ldots,L$, $\varphi=1,\ldots,\Psi^1_{l(h)}$, $n_{l(h)}>0$,

$$\left[\rho\sum_{i=1}^{s}a_i+\mu_{l(h)}^{1,\varphi}\right]p_r(\boldsymbol{n},h,1,\varphi)=\rho\sum_{i=1}^{s}a_i p_r(\boldsymbol{n}-\boldsymbol{e}_i,h,1,\varphi)I\{n_i>0\}$$
$$+\ \mu_{l(h)}^{1,\varphi+1}p_r(\boldsymbol{n},h,1,\varphi+1)I\{\varphi<\Psi_{l(h)}^1\}\ +\ \mu_h^{0,1}\pi_{l(h)}^{1,\varphi}p_r(\boldsymbol{n},h,0,1) \tag{17}$$
$$+\ q_h\mu_{l(h)}^{1,1}\pi_{l(h)}^{1,\varphi}p_r(\boldsymbol{n}+\boldsymbol{e}_{l(h)},h,1,1)\ +\ \mu_{l(h)}^{1,1}\pi_{l(h)}^{1,\varphi}p(\boldsymbol{n}+\boldsymbol{e}_{l(h)},h,1,1).$$

From (13) and (15) it follows that for all $h=1,\ldots,L$, $\varphi=1,\ldots,\Psi^1_{l(h)}$, $r=1,\ldots,L$,

$$p_r(\boldsymbol{n},h,1,\varphi)=0,\quad \text{if}\quad n_{l(h)}=0. \tag{18}$$

Moreover, from (14) we have for $r=1,\ldots,L$,

$$\sum_{(\boldsymbol{n},h,\zeta,\varphi)\in\mathbf{N}^s\times\Theta} p_r(\boldsymbol{n},h,\zeta,\varphi)\ =\ 0. \tag{19}$$

Remarks:

When some of the switch-over times are zero, some straightforward modifications of (16) and (17) have to be made.

This also opens the possibility of calculating the performance of polling systems with limited service disciplines by means of the p.s.a.. In order to model a K-limited service discipline at some queue, the polling table should prescribe K successive visits to that particular queue; during each of these visits the queue should be served 1-limited (i.e., $q_h=0$) and the switch-over times between these successive visits should be zero.

4. The computation scheme

The basic idea of the p.s.a. is the transformation of the non-recursively solvable infinite set of balance equations into an, in principle, recursively solvable (infinite) set of equations by adding one dimension to the state space. To this end, the state-probabilities and their derivatives with respect to the Bernoulli parameters are expressed as power-series in terms of the offered load ρ. A complete recursive scheme to compute all coefficients for the state-probabilities has been derived in [4]. Following the same approach, in this section we shall derive a complete recursive scheme to compute the coefficients of the power-series expansions of the derivatives of the state-probabilities with respect to the Bernoulli parameters.

The approach in [4] relies on the following property:

$$p(\mathbf{n},h,\zeta,\varphi)=O(\rho^{n_1+\dots+n_s}),\quad (\rho\downarrow 0), \tag{20}$$

for $(\mathbf{n}, h, \zeta, \varphi)\in\mathbf{N}^s\times\Theta$. Based on this property, introduce the following power-series expansions for the state-probabilities:

$$p(\mathbf{n},h,\zeta,\varphi)=\rho^{n_1+\dots+n_s}\sum_{k=0}^{\infty}\rho^k b_0(k;\mathbf{n},h,\zeta,\varphi), \tag{21}$$

for $(\mathbf{n}, h, \zeta, \varphi)\in\mathbf{N}^s\times\Theta$. In this same way, for $r=1,\dots,L$, we introduce the power-series expansions

$$p_r(\mathbf{n},h,\zeta,\varphi)=\rho^{n_1+\dots+n_s}\sum_{k=0}^{\infty}\rho^k b_r(k;\mathbf{n},h,\zeta,\varphi); \tag{22}$$

because ρ does not depend on the value of $\mathbf{q}$, we have for all $(k; \mathbf{n}, h, \zeta, \varphi)\in\mathbf{N}^{1+s}\times\Theta$:

$$b_r(k;\mathbf{n},h,\zeta,\varphi)=\frac{\partial}{\partial q_r}b_0(k;\mathbf{n},h,\zeta,\varphi). \tag{23}$$

Note that the mean number of customers at queue i, $i=1,\dots,s$, and its derivative with respect to q_r, $r=1,\dots,L$, can be expressed in terms of these coefficients as

$$EN_i=\sum_{k=0}^{\infty}\rho^k\sum_{0<n_1+\dots+n_s\le k}\ \sum_{(h,\zeta,\varphi)\in\Theta}n_i b_0(k-n_1-\dots-n_s;\mathbf{n},h,\zeta,\varphi) \tag{24}$$

and

$$\frac{\partial}{\partial q_r} EN_i = \sum_{k=0}^{\infty} \rho^k \sum_{0<n_1+\ldots+n_s\le k} \sum_{(h,\zeta,\varphi)\in\Theta} n_i b_r(k-n_1-\ldots-n_s;\mathbf{n},h,\zeta,\varphi) \tag{25}$$

respectively. The mean waiting time at queue i, i = 1, . . . , s, and its derivative with respect to q_r, r = 1, . . . , L, can be obtained by using Little's formula.

Substituting (21) and (22) into the balance equations (11), (12) and (16), (17) respectively, and equating the corresponding coefficients of powers of ρ leads to the following set of equations for the coefficients, for k = 0, 1, . . . , $\mathbf{n}\in\mathbf{N}^s$, h = 1, . . . , L, φ = 1, . . . , Ψ_h^0, r = 0, . . . , L,

$$\begin{aligned}
\mu_h^{0,\varphi} b_r(k;\mathbf{n},h,0,\varphi) &= \mu_h^{0,\varphi+1} b_r(k;\mathbf{n},h,0,\varphi+1) I\{\varphi<\Psi_h^0\} \\
&+ \sum_{i=1}^{s} a_i \big[b_r(k;\mathbf{n}-e_i,h,0,\varphi) I\{n_i>0\} - b_r(k-1;\mathbf{n},h,0,\varphi) I\{k>0\} \big] \\
&+ \mu_{I(h-1)}^{1,1} \pi_h^{0,\varphi} b_r(k-1;\mathbf{n}+e_{I(h-1)},h-1,1,1) I\{k>0\} \big[1-q_{h-1} I\{n_{I(h-1)}>0\}\big] \\
&+ \mu_{I(h-1)}^{0,1} \pi_h^{0,\varphi} b_r(k;\mathbf{n},h-1,0,1) I\{n_{I(h-1)}=0\} \\
&- \mu_{I(h-1)}^{1,1} \pi_h^{0,\varphi} b_0(k-1;\mathbf{n}+e_{I(h-1)},h-1,1,1) I\{k>0\} I\{r>0\} I\{n_{I(h-1)}>0\};
\end{aligned} \tag{26}$$

and for k = 0, 1, . . . , $\mathbf{n}\in\mathbf{N}^s$, h = 1, . . . , L, φ = 1, . . . , $\Psi_{I(h)}^1$, $n_{I(h)}>0$, r = 0, . . . , L,

$$\begin{aligned}
\mu_{I(h)}^{1,\varphi} b_r(k;\mathbf{n},h,1,\varphi) &= \mu_{I(h)}^{1,\varphi+1} b_r(k;\mathbf{n},h,1,\varphi+1) I\{\varphi<\Psi_{I(h)}^1\} \\
&+ \mu_h^{0,1} \pi_{I(h)}^{1,\varphi} b_r(k;\mathbf{n},h,0,1) + q_h \mu_{I(h)}^{1,1} \pi_{I(h)}^{1,\varphi} b_r(k-1;\mathbf{n}+e_{I(h)},h,1,1) I\{k>0\} \\
&+ \sum_{i=1}^{s} a_i \big[b_r(k;\mathbf{n}-e_i,h,1,\varphi) I\{n_i>0\} - b_r(k-1;\mathbf{n},h,1,\varphi) I\{k>0\} \big] \\
&+ \mu_{I(h)}^{1,1} \pi_{I(h)}^{1,\varphi} b_0(k-1;\mathbf{n}+e_{I(h)},h,1,1) I\{k>0\} I\{r>0\}.
\end{aligned} \tag{27}$$

We extend the partial ordering ≺ of the states (k; **n**, h, ζ, φ) as in [4] to the following partial ordering ≺≺ of the vectors (r, k; **n**, h, ζ, φ) ∈ {0, . . . , L} × $\mathbf{N}^{1+s}$ × Θ:

$$
\begin{aligned}
(r_1,k_1;\mathbf{m},h_1,\zeta_1,\varphi_1) \prec\prec (r_2,k_2;\mathbf{n},h_2,\zeta_2,\varphi_2)& \\
\textit{if } [r_1=0 \textit{ and } r_2>0]& \\
\textit{or if } [r_1=r_2 \textit{ and } (k_1;\mathbf{m},h_1,\zeta_1,\varphi_1) \prec (k_2;\mathbf{n},h_2,\zeta_2,\varphi_2)].&
\end{aligned}
\tag{28}
$$

Under the ordering $\prec\prec$ this set of equations (26), (27) forms a recursive scheme for all coefficients $b_r(k;\ \mathbf{n},\ h,\ \zeta,\ \varphi)$, $(k;\ \mathbf{n},\ h,\ \zeta,\ \varphi)\in\mathbb{N}^{1+s}\times\Theta$, $r=0, \ldots, L$, except for those states with $\mathbf{n}=\mathbf{0}$; this is because equations (26) and (27) express the coefficients $b_r(k;\ \mathbf{n},\ h,\ \zeta,\ \varphi)$ in terms of coefficients with a lower order with respect to $\prec\prec$. Hence, the only states which require further attention are the states with $\mathbf{n}=\mathbf{0}$ and $\zeta=0$. For these states, equations (26) read for $r=0, \ldots, L$, $k=0, 1, \ldots,$ $h=1, \ldots, L$, $\varphi=1, \ldots \Psi_h^0$,

$$
\begin{aligned}
\mu_h^{0,\varphi} b_r(k;\mathbf{0},h,0,\varphi) = {}& \mu_h^{0,\varphi+1} b_r(k;\mathbf{0},h,0,\varphi+1) I\{\varphi<\Psi_h^0\} \\
&+ \mu_{h-1}^{0,1}\pi_h^{0,\varphi} b_r(k;\mathbf{0},h-1,0,1) + y_r(k;h,\varphi);
\end{aligned}
\tag{29}
$$

here, the quantities $y_r(k;\ h,\ \varphi)$, $r=0, \ldots, L$, $h=1, \ldots, L$, $\varphi=1, \ldots, \Psi_h^0$, defined by $y_r(0;\ h,\ \varphi):=0$ and for $k=1, 2, \ldots$ by

$$
\begin{aligned}
y_r(k;h,\varphi) := {}& \mu_{l(h-1)}^{1,1}\pi_h^{0,\varphi} b_r(k-1;\mathbf{e}_{l(h-1)},h-1,1,1) \\
&- \sum_{i=1}^{s} a_i b_r(k-1;\mathbf{0},h,0,\varphi),
\end{aligned}
\tag{30}
$$

consist of terms with coefficients of lower order with respect to $\prec\prec$ than $(r,\ k;\ \mathbf{0},\ h,\ 0,\ \varphi)$, and hence, can be considered to be known. For k fixed, the sets of equations (29) are dependent. From (15) and (20) we have

$$
\sum_{h=1}^{L}\sum_{\varphi=1}^{\Psi_h^0} b_0(0;\mathbf{0},h,0,\varphi) = I\{r=0\},
\tag{31}
$$

and moreover, for $k=1, 2, \ldots,$ $r=0, \ldots, L$,

$$
\sum_{h=1}^{L}\sum_{\varphi=1}^{\Psi_h^0} b_r(k;\mathbf{0},h,0,\varphi) = Y_r(k),
\tag{32}
$$

with

$$Y_r(k) := \sum_{0<n_1+\ldots+n_s\le k} \; \sum_{(h,\zeta,\varphi)\in\Theta} b_r(k-n_1-\ldots-n_s;\mathbf{n},h,\zeta,\varphi). \tag{33}$$

Consider, for k fixed, the set of equations consisting of all but one of the equations (29) combined with either (31) or (32). Then the determinants $\Delta(k)$ of these sets of equations are independent of k and are given by

$$\Delta(k) = \Delta := \sigma_1 \prod_{h=1}^{L} \prod_{\varphi=1}^{\Psi_h^0} \mu_h^{0,\varphi}. \tag{34}$$

For $k=0$, this set of equations is readily solved: for $h=1, \ldots, L$, $\varphi=1, \ldots, \Psi_h^0$,

$$b_r(0;\mathbf{0},h,0,\varphi) = \begin{cases} \dfrac{1}{\sigma_1 \mu_h^{0,\varphi}} \displaystyle\sum_{\psi=\varphi}^{\Psi_h^0} \pi_h^{0,\psi} & \text{if } r=0, \\[2ex] 0 & \text{if } r>0. \end{cases} \tag{35}$$

It is tedious but straightforward to show that for $k=1, 2, \ldots$, $r=0, \ldots, L$,

$$b_r(k;\mathbf{0},L,0,1) = \frac{-1}{\sigma_1 \mu_L^{0,1}} \left[Y_r(k) + \sum_{h=1}^{L} \sum_{\varphi=1}^{\Psi_h^0} \frac{1}{\mu_h^{0,\varphi}} \sum_{\psi=\varphi}^{\Psi_h^0} \left\{ y_r(k;h,\psi) + \pi_h^{0,\psi} \sum_{j=1}^{h-1} \sum_{v=1}^{\Psi_j^0} y_r(k;j,v) \right\} \right]. \tag{36}$$

Summarizing, all coefficients $b_r(k;\, \mathbf{n},\, h,\, \zeta,\, \varphi)$, $(k;\, \mathbf{n},\, h,\, \zeta,\, \varphi)\in\mathbb{N}^{1+s}\times\Theta$, $r=0, \ldots, L$, can, in principle, be recursively computed using the following computation scheme:

step 0: determine $b_r(0;\, \mathbf{0},\, h,\, 0,\, \varphi)$, $r=0, \ldots, L$, $h=1, \ldots, L$, $\varphi=1, \ldots, \Psi_h^0$, according to (35);

step 1: $M:=1$;

step 2: for all $(k;\, \mathbf{n},\, h,\, \zeta,\, \varphi)\in\mathbb{N}^{1+s}\times\Theta$, with $(\mathbf{n},\zeta)\neq(0,0)$ and with $k + n_1 + \ldots + n_s = M$, determine $b_r(k;\, \mathbf{n},\, h,\, \zeta,\, \varphi)$, according to (26) and (27), in increasing order of $(r,\, k;\, \mathbf{n},\, h,\, \zeta,\, \varphi)$ with respect to $\prec\prec$;

step 3: determine $b_r(M;\, \mathbf{0},\, L,\, 0,\, 1)$, $r=0, \ldots, L$, according to (36);

step 4: compute $b_r(M; \mathbf{0}, h, 0, \varphi)$, $r=0, \ldots, L$, $h=1, \ldots, L$, $\varphi=1, \ldots, \Psi_h^0$, according to (29) and (30); the order of the couples (h,φ) is $(1,\Psi_h^0), \ldots, (1,1), \ldots, (L,\Psi_h^0), \ldots, (L,2)$;

step 5: $M:=M+1$; repeat from step 2.

Remarks:

The recursive scheme provides a means to evaluate all state-probabilities and, in particular, the mean number of customers at each of the queues as well as their derivatives with respect to the Bernoulli parameters according to (24) and (25) up to, in principle, any level of accuracy.

From (26) and (27) it follows that the coefficients $b_r(k; \mathbf{n}, h, \zeta, \varphi)$, $r=1, \ldots, L$, cannot be computed when $b_0(k; \mathbf{n}, h, \zeta, \varphi)$ is unknown, but that the partial derivatives can be computed independently of each other, so that it is possible to compute derivatives with respect to q_r for r in some subset D of $\{1, \ldots, L\}$. Then it is most appropriate to compute for each $(k; \mathbf{n}, h, \zeta, \varphi)$ the coefficients $b_r(k; \mathbf{n}, h, \zeta, \varphi)$, $r\in\{0\} \cup D$ successively, before computing terms of higher order with respect to $\prec\prec$, because in this way the coefficients $b_0(k; \mathbf{n}, h, \zeta, \varphi)$ have to be computed only once.

For the model without switch-over times, the presence of a unique zero state leads to a different set of balance equations, cf. [3] for the special case with exponentially distributed service times. A computation scheme for the partial derivatives can be obtained in a similar way as has been done in this section.

Higher order derivatives with respect to the Bernoulli parameters can, in principle, be computed by means of the p.s.a. following the same approach as discussed in Sections 3 and 4.

5. Efficient use of memory space and computation time

The main restriction in applying the p.s.a. is the fact that the time and memory requirements increase exponentially with the number of queues (cf. [4]), so that the use is restricted to moderately sized systems. Moreover, one may verify from the previous section that when the p.s.a. is applied to compute first order derivatives the computational burden increases linearly with the number of partial derivatives which have to be computed. In this section we shall discuss a few ways to reduce the required amount of memory space and computation time for the use of the p.s.a. for the present model with partial derivatives.

A first way to save memory space and computation time is to reduce the dimension of the optimization problem in cases in which one or more components of the optimal schedule are known or have to be kept fixed for some reason. In fact, numerical experience has suggested the following property, which is supported by results in [12]:

(i) *if* $c_i/\rho_i = \max_{j=1,\ldots,s}\{c_j/\rho_j\}$, *then* $q_h^* = 1$ *for all* h *for which* $l(h) = i$; (37)

(ii) *if* $\sigma_1 = 0$, *then if* $c_i/\rho_i = \min_{j=1,\ldots,s}\{c_j/\rho_j\}$, *then* $q_h^* = 0$ *for all* h *for which* $l(h) = i$. (38)

Another way to limit the time and memory requirements is the reduction of the number of terms that has to be computed by applying an algorithm to accelerate the convergence, such as the ε-algorithm. The ε-algorithm aims to accelerate the convergence of slowly convergent sequences or to determine a value for divergent sequences (cf. [17, 6]). When the heavy-traffic behaviour of the performance measures is known beforehand, the performance of the ε-algorithm can be considerably improved by a modification of the initial values of the ε-algorithm (cf. [4]). In particular, the coefficients of the power-series are extrapolated to take into account the pole of the moments at $\rho = \rho_0$. It should be noted that not all queues become instable when $\rho \uparrow \rho_0$; more precisely, the set of queues that become instable as $\rho \uparrow \rho_0$ is given by (cf. [7])

$$R := \{1 \le i \le s \mid a_i/M_i = \max_{j=1,\ldots,s} a_j/M_j\}. \tag{39}$$

For the heavy-traffic behaviour of the mean waiting times and their partial derivatives with respect to the Bernoulli parameters we have found the following two properties for $i = 1, \ldots, s$, $r = 1, \ldots, L$:

1) if $i \in R$, then EW_i has a pole of order 1 at $\rho = \rho_0$;
otherwise, EW_i has a finite limit for $\rho \uparrow \rho_0$. (40)

2) if ($\sigma_1 = 0$ and $\#R \ge 2$) or if $\sigma_1 > 0$, then

if $i \in R$ and $l(r) \in R$, then $\frac{\partial}{\partial q_r} EW_i$ *has a pole of order 2 at $\rho = \rho_0$;*

in all other cases, $\frac{\partial}{\partial q_r} EW_i$ *has a finite limit for $\rho \uparrow \rho_0$.* (41)

Using these properties for the initial values in the ε-algorithm has led to a considerable improvement of the convergence of the p.s.a. for $\rho \uparrow \rho_0$.

Moreover, knowing some of the coefficients beforehand may lead to considerable savings of the required memory space. In fact, from relations (35), (26) and (27), it follows that $b_r(0; \mathbf{n}, h, \zeta, \varphi)=0$ for all $(\mathbf{n}, h, \zeta, \varphi)\in \mathbf{N}^s\times\Theta$, $r=1, \ldots, L$, and moreover, with (36), $b_r(1; \mathbf{0}, h, \zeta, \varphi)=0$, for all $(h, \zeta, \varphi)\in\Theta$, $r=1, \ldots, L$. Suppose the coefficients of the state-probabilities have to be computed up to the M-th power of ρ. Then, from the definition of the ordering $\prec\prec$, the coefficients $b_r(k; \mathbf{n}, h, \zeta, \varphi)$, $(h,\zeta,\varphi)\in\Theta$, $r\in\{0\} \cup D$, with $k + n_1 + \ldots + n_s \le M$, have to be computed. One may verify that the number of coefficients that have to be computed is reduced to (cf. [2])

$$\left[(|D|+1) \times \binom{M+s+1}{s+1} - |D| \times \binom{M+s}{s} - |D| \right] \times \sum_{h=1}^{L} \left\{ \Psi_h^0 + \Psi_{l(h)}^1 \right\}, \tag{42}$$

i.e., the number of triples $(h, \zeta, \varphi)\in\Theta$, for which there exists $\mathbf{n}\in\mathbf{N}^s$ such that the states $(\mathbf{n}, h, \zeta, \varphi)$ can actually be entered.

In practice, this reduction leads to a slight increase of the number of coefficients that can be computed for a given amount of available memory space.

The reader is referred to [2] (Section 5) for some ideas for an efficient implementation of the p.s.a. which are useful in a more general context than for the present model.

6. Comparison

In order to numerically solve the optimization problem defined in Section 2, in [5] we proposed (for the special case of cyclic polling systems) to use the p.s.a. combined with a local optimization procedure, such as the conjugate gradient method. Using this approach partial derivatives which are required for this method were estimated on the basis of finite differences; the problem here was the fact that these differences could only be estimated based on the last few computed terms of the power-series. Moreover, neighbouring schedules had to be evaluated to estimate the partial derivatives. To overcome these difficulties, in this paper we extended the use of the p.s.a. towards the evaluation of first order partial derivatives with respect to the Bernoulli parameters.

Therefore, in this section we make a comparison between (i) the estimation approach, in which partial derivatives are estimated based on finite differences (cf. [5]), and (ii) the computation approach, in which partial derivatives are computed by means of the p.s.a. (cf. Sections 3 and 4).

Numerical experience has taught us that using the computation approach the number of search directions needed to find an optimal schedule is, in general, considerably smaller compared to the estimation approach, especially for higher dimensional optimization problems. This may be due to the fact that using the computation approach search directions can be calculated accurately by means of the p.s.a. rather than estimating search directions based on a fairly arbitrarily chosen step size.
Moreover, the number of schedules that have to be evaluated to find an optimum is strongly reduced when the computation approach is used. This is because, apart from the fact that the number of search directions is smaller, no neighbouring schedules have to be evaluated in order to estimate search directions.
As noted in the previous section, the time requirements of using the computation approach increase linearly with the dimension of the optimization problem. However, the effect of this increase in time requirements is *dominated* by the savings in computation time of the decrease in the number of schedules and search directions that have to be computed. In fact, numerous numerical experiments have indicated that the total amount of computation time is decreased by at least a factor two, while this factor increases when the dimension of the optimization problem is increased.

A main restriction of the use of the p.s.a. is the required amount of memory space, which may lead to a considerable reduction in the number of terms of the power-series that can be computed and hence, in the accuracy of the computations. As memory requirements increase with the number of partial derivatives, the maximal number of terms that can be computed using the estimation approach (in which only the coefficients $b_0(k; \mathbf{n}, h, \zeta, \boldsymbol{\varphi})$ have to be calculated) may be considerably larger compared with the computation approach (in which the coefficients $b_r(k; \mathbf{n}, h, \zeta, \boldsymbol{\varphi})$, $r=0, \ldots, L$, have to be calculated), generally leading to a more accurate determination of the optimal schedule. This difference in accuracy becomes apparent in higher dimensional optimization problems.
Alternatively, in these cases one may also partition the set D of components of the Bernoulli schedule for which an optimal value has to be determined into proper subsets, say $D_1, \ldots, D_t$ $(t>1)$, and compute successively the coefficients $b_r(k; \mathbf{n}, h, \zeta, \boldsymbol{\varphi})$, $(k; \mathbf{n}, h, \zeta, \boldsymbol{\varphi}) \in \mathbb{N}^{1+s} \times \Theta$, $r \in \{0\} \cup D_j$, where j is varied from 1 to t (recall the second remark in Section 4). In this way, an extra number of terms of the power-series can be computed, leading to more accurate evaluations; this goes at the expense of computation time, because of the fact that the coefficients $b_0(k; \mathbf{n}, h, \zeta, \boldsymbol{\varphi})$) have to be computed t times instead of once.

Remark:

In practice, it is useful to apply the p.s.a. with a small number of terms to find the neighbourhood of the optimal schedule with reduced computational effort, and then proceed with the p.s.a. with more terms to locally improve the optimal schedule.

Acknowledgement: The authors wish to thank Onno Boxma for interesting discussions and useful comments.

References

[1] J.P.C. Blanc, On a numerical method for calculating state-probabilities for queueing systems with more than one waiting line, *Journal of Computational and Applied Mathematics* **20** (1987) 119-125.

[2] J.P.C. Blanc, A numerical approach to cyclic-service queueing models, *Queueing Systems* **6** (1990) 173-188.

[3] J.P.C. Blanc, An algorithmic solution of polling systems with limited service disciplines, *IEEE Transactions on Communications* **40** (1992) 1152-1155.

[4] J.P.C. Blanc, Performance evaluation of polling systems by means of the power-series algorithm, *Annals of Operations Research* **35** (1992) 155-186.

[5] J.P.C. Blanc and R.D. van der Mei, Optimization of polling systems with Bernoulli schedules, *Report FEW 563, Department of Economics, Tilburg University, Tilburg* (1992).

[6] C. Brezinsky, Accélération de la Convergence en Analyse Numérique, *Lecture Notes in Mathematics* **584** (Springer, Heidelberg) (1977).

[7] C. Fricker and M.R. Jaïbi, Monotonicity and stability of periodic polling models, *Report FEW 559, Department of Economics, Tilburg University, Tilburg* (1992).

[8] G. Hooghiemstra, M.S. Keane and S. van der Ree, Power series for stationary distributions of coupled processor models, *SIAM Journal of Applied Mathematics* **48** (1988) 1159-1166.

[9] J. Keilson and L.D. Servi, Oscillating random walk models for GI/G/1 vacation systems with Bernoulli schedules, *Journal of Applied Probability* **23** (1986) 790-802.

[10] K.K. Leung, Cyclic-service systems with probabilistically-limited service, *IEEE Selected Areas in Communications* **9** (1991) 185-193.

[11] H. Levy and M. Sidi, Polling systems: applications, modeling and optimization, *IEEE Transactions on Communications* **38** (1990) 1750-1760.

[12] I. Meilijson and U. Yechiali, On optimal right-of-way policies at a single-server station when insertion of idle times is permitted, *Stochastic Processes and their Applications* **6** (1977) 23-52.

[13] S.S. Rao, Optimization Theory and Applications (Wiley Eastern Limited, New Delhi) (1978).

[14] H. Takagi, Queueing analysis of polling models, in: *Stochastic Analysis of Computer and Communication Systems, ed. H.Takagi* (North-Holland, Amsterdam) (1990) 267-318.

[15] T.E. Tedijanto, Exact results for the cyclic-service queue with a Bernoulli schedule, *Performance Evaluation* **11** (1990) 107-115.

[16] J.A. Weststrate, Waiting times in a two-queue model with exhaustive and Bernoulli service, *Report FEW 437, Department of Economics, Tilburg University, Tilburg* (1990). To appear in: *Z.O.R. Methods and Models of Operations Research.*

[17] P. Wynn, On the convergence and stability of the epsilon algorithm, *SIAM Journal of Numerical Analysis* **3** (1966) 91-122.

IN 1991 REEDS VERSCHENEN

466 Prof.Dr. Th.C.M.J. van de Klundert - Prof.Dr. A.B.T.M. van Schaik
Economische groei in Nederland in een internationaal perspectief

467 Dr. Sylvester C.W. Eijffinger
The convergence of monetary policy - Germany and France as an example

468 E. Nijssen
Strategisch gedrag, planning en prestatie. Een inductieve studie binnen de computerbranche

469 Anne van den Nouweland, Peter Borm, Guillermo Owen and Stef Tijs
Cost allocation and communication

470 Drs. J. Grazell en Drs. C.H. Veld
Motieven voor de uitgifte van converteerbare obligatieleningen en warrant-obligatieleningen: een agency-theoretische benadering

471 P.C. van Batenburg, J. Kriens, W.M. Lammerts van Bueren and R.H. Veenstra
Audit Assurance Model and Bayesian Discovery Sampling

472 Marcel Kerkhofs
Identification and Estimation of Household Production Models

473 Robert P. Gilles, Guillermo Owen, René van den Brink
Games with Permission Structures: The Conjunctive Approach

474 Jack P.C. Kleijnen
Sensitivity Analysis of Simulation Experiments: Tutorial on Regression Analysis and Statistical Design

475 C.P.M. van Hoesel
An $O(nlogn)$ algorithm for the two-machine flow shop problem with controllable machine speeds

476 Stephan G. Vanneste
A Markov Model for Opportunity Maintenance

477 F.A. van der Duyn Schouten, M.J.G. van Eijs, R.M.J. Heuts
Coordinated replenishment systems with discount opportunities

478 A. van den Nouweland, J. Potters, S. Tijs and J. Zarzuelo
Cores and related solution concepts for multi-choice games

479 Drs. C.H. Veld
Warrant pricing: a review of theoretical and empirical research

480 E. Nijssen
De Miles and Snow-typologie: Een exploratieve studie in de meubelbranche

481 Harry G. Barkema
Are managers indeed motivated by their bonuses?

482 Jacob C. Engwerda, André C.M. Ran, Arie L. Rijkeboer
Necessary and sufficient conditions for the existence of a positive definite solution of the matrix equation $X + A^T X^{-1} A = I$

483 Peter M. Kort
A dynamic model of the firm with uncertain earnings and adjustment costs

484 Raymond H.J.M. Gradus, Peter M. Kort
Optimal taxation on profit and pollution within a macroeconomic framework

485 René van den Brink, Robert P. Gilles
Axiomatizations of the Conjunctive Permission Value for Games with Permission Structures

486 A.E. Brouwer & W.H. Haemers
The Gewirtz graph - an exercise in the theory of graph spectra

487 Pim Adang, Bertrand Melenberg
Intratemporal uncertainty in the multi-good life cycle consumption model: motivation and application

488 J.H.J. Roemen
The long term elasticity of the milk supply with respect to the milk price in the Netherlands in the period 1969-1984

489 Herbert Hamers
The Shapley-Entrance Game

490 Rezaul Kabir and Theo Vermaelen
Insider trading restrictions and the stock market

491 Piet A. Verheyen
The economic explanation of the jump of the co-state variable

492 Drs. F.L.J.W. Manders en Dr. J.A.C. de Haan
De organisatorische aspecten bij systeemontwikkeling
een beschouwing op besturing en verandering

493 Paul C. van Batenburg and J. Kriens
Applications of statistical methods and techniques to auditing and accounting

494 Ruud T. Frambach
The diffusion of innovations: the influence of supply-side factors

495 J.H.J. Roemen
A decision rule for the (des)investments in the dairy cow stock

496 Hans Kremers and Dolf Talman
An SLSPP-algorithm to compute an equilibrium in an economy with linear production technologies

497 L.W.G. Strijbosch and R.M.J. Heuts
Investigating several alternatives for estimating the compound lead time demand in an (s,Q) inventory model

498 Bert Bettonvil and Jack P.C. Kleijnen
Identifying the important factors in simulation models with many factors

499 Drs. H.C.A. Roest, Drs. F.L. Tijssen
Beheersing van het kwaliteitsperceptieproces bij diensten door middel van keurmerken

500 B.B. van der Genugten
Density of the F-statistic in the linear model with arbitrarily normal distributed errors

501 Harry Barkema and Sytse Douma
The direction, mode and location of corporate expansions

502 Gert Nieuwenhuis
Bridging the gap between a stationary point process and its Palm distribution

503 Chris Veld
Motives for the use of equity-warrants by Dutch companies

504 Pieter K. Jagersma
Een etiologie van horizontale internationale ondernemingsexpansie

505 B. Kaper
On M-functions and their application to input-output models

506 A.B.T.M. van Schaik
Produktiviteit en Arbeidsparticipatie

507 Peter Borm, Anne van den Nouweland and Stef Tijs
Cooperation and communication restrictions: a survey

508 Willy Spanjers, Robert P. Gilles, Pieter H.M. Ruys
Hierarchical trade and downstream information

509 Martijn P. Tummers
The Effect of Systematic Misperception of Income on the Subjective Poverty Line

510 A.G. de Kok
Basics of Inventory Management: Part 1
Renewal theoretic background

511 J.P.C. Blanc, F.A. van der Duyn Schouten, B. Pourbabai
Optimizing flow rates in a queueing network with side constraints

512 R. Peeters
On Coloring j-Unit Sphere Graphs

513 Drs. J. Dagevos, Drs. L. Oerlemans, Dr. F. Boekema
Regional economic policy, economic technological innovation and networks

514 Erwin van der Krabben
Het functioneren van stedelijke onroerend-goed-markten in Nederland - een theoretisch kader

515 Drs. E. Schaling
European central bank independence and inflation persistence

516 Peter M. Kort
Optimal abatement policies within a stochastic dynamic model of the firm

517 Pim Adang
Expenditure versus consumption in the multi-good life cycle consumption model

518 Pim Adang
Large, infrequent consumption in the multi-good life cycle consumption model

519 Raymond Gradus, Sjak Smulders
Pollution and Endogenous Growth

520 Raymond Gradus en Hugo Keuzenkamp
Arbeidsongeschiktheid, subjectief ziektegevoel en collectief belang

521 A.G. de Kok
Basics of inventory management: Part 2
The (R,S)-model

522 A.G. de Kok
Basics of inventory management: Part 3
The (b,Q)-model

523 A.G. de Kok
Basics of inventory management: Part 4
The (s,S)-model

524 A.G. de Kok
Basics of inventory management: Part 5
The (R,b,Q)-model

525 A.G. de Kok
Basics of inventory management: Part 6
The (R,s,S)-model

526 Rob de Groof and Martin van Tuijl
Financial integration and fiscal policy in interdependent two-sector economies with real and nominal wage rigidity

527 A.G.M. van Eijs, M.J.G. van Eijs, R.M.J. Heuts
Gecoördineerde bestelsystemen
een management-georiënteerde benadering

528 M.J.G. van Eijs
Multi-item inventory systems with joint ordering and transportation decisions

529 Stephan G. Vanneste
Maintenance optimization of a production system with buffercapacity

530 Michel R.R. van Bremen, Jeroen C.G. Zijlstra
Het stochastische variantie optiewaarderingsmodel

531 Willy Spanjers
Arbitrage and Walrasian Equilibrium in Economies with Limited Information

IN 1992 REEDS VERSCHENEN

532 F.G. van den Heuvel en M.R.M. Turlings
Privatisering van arbeidsongeschiktheidsregelingen
Refereed by Prof.Dr. H. Verbon

533 J.C. Engwerda, L.G. van Willigenburg
LQ-control of sampled continuous-time systems
Refereed by Prof.dr. J.M. Schumacher

534 J.C. Engwerda, A.C.M. Ran & A.L. Rijkeboer
Necessary and sufficient conditions for the existence of a positive definite solution of the matrix equation $X + A^*X^{-1}A = Q$.
Refereed by Prof.dr. J.M. Schumacher

535 Jacob C. Engwerda
The indefinite LQ-problem: the finite planning horizon case
Refereed by Prof.dr. J.M. Schumacher

536 Gert-Jan Otten, Peter Borm, Ton Storcken, Stef Tijs
Effectivity functions and associated claim game correspondences
Refereed by Prof.dr. P.H.M. Ruys

537 Jack P.C. Kleijnen, Gustav A. Alink
Validation of simulation models: mine-hunting case-study
Refereed by Prof.dr.ir. C.A.T. Takkenberg

538 V. Feltkamp and A. van den Nouweland
Controlled Communication Networks
Refereed by Prof.dr. S.H. Tijs

539 A. van Schaik
Productivity, Labour Force Participation and the Solow Growth Model
Refereed by Prof.dr. Th.C.M.J. van de Klundert

540 J.J.G. Lemmen and S.C.W. Eijffinger
The Degree of Financial Integration in the European Community
Refereed by Prof.dr. A.B.T.M. van Schaik

541 J. Bell, P.K. Jagersma
Internationale Joint Ventures
Refereed by Prof.dr. H.G. Barkema

542 Jack P.C. Kleijnen
Verification and validation of simulation models
Refereed by Prof.dr.ir. C.A.T. Takkenberg

543 Gert Nieuwenhuis
Uniform Approximations of the Stationary and Palm Distributions of Marked Point Processes
Refereed by Prof.dr. B.B. van der Genugten

544 R. Heuts, P. Nederstigt, W. Roebroek, W. Selen
Multi-Product Cycling with Packaging in the Process Industry
Refereed by Prof.dr. F.A. van der Duyn Schouten

545 J.C. Engwerda
Calculation of an approximate solution of the infinite time-varying LQ-problem
Refereed by Prof.dr. J.M. Schumacher

546 Raymond H.J.M. Gradus and Peter M. Kort
On time-inconsistency and pollution control: a macroeconomic approach
Refereed by Prof.dr. A.J. de Zeeuw

547 Drs. Dolph Cantrijn en Dr. Rezaul Kabir
De Invloed van de Invoering van Preferente Beschermingsaandelen op Aandelenkoersen van Nederlandse Beursgenoteerde Ondernemingen
Refereed by Prof.dr. P.W. Moerland

548 Sylvester Eijffinger and Eric Schaling
Central bank independence: criteria and indices
Refereed by Prof.dr. J.J. Sijben

549 Drs. A. Schmeits
Geïntegreerde investerings- en financieringsbeslissingen; Implicaties voor Capital Budgeting
Refereed by Prof.dr. P.W. Moerland

550 Peter M. Kort
Standards versus standards: the effects of different pollution restrictions on the firm's dynamic investment policy
Refereed by Prof.dr. F.A. van der Duyn Schouten

551 Niels G. Noorderhaven, Bart Nooteboom and Johannes Berger
Temporal, cognitive and behavioral dimensions of transaction costs; to an understanding of hybrid vertical inter-firm relations
Refereed by Prof.dr. S.W. Douma

552 Ton Storcken and Harrie de Swart
Towards an axiomatization of orderings
Refereed by Prof.dr. P.H.M. Ruys

553 J.H.J. Roemen
The derivation of a long term milk supply model from an optimization model
Refereed by Prof.dr. F.A. van der Duyn Schouten

554 Geert J. Almekinders and Sylvester C.W. Eijffinger
Daily Bundesbank and Federal Reserve Intervention and the Conditional Variance Tale in DM/$-Returns
Refereed by Prof.dr. A.B.T.M. van Schaik

555 Dr. M. Hetebrij, Drs. B.F.L. Jonker, Prof.dr. W.H.J. de Freytas
"Tussen achterstand en voorsprong" de scholings- en personeelsvoorzieningsproblematiek van bedrijven in de procesindustrie
Refereed by Prof.dr. Th.M.M. Verhallen

556 Ton Geerts
Regularity and singularity in linear-quadratic control subject to implicit continuous-time systems
Communicated by Prof.dr. J. Schumacher

557 Ton Geerts
Invariant subspaces and invertibility properties for singular systems: the general case
Communicated by Prof.dr. J. Schumacher

558 Ton Geerts
Solvability conditions, consistency and weak consistency for linear differential-algebraic equations and time-invariant singular systems: the general case
Communicated by Prof.dr. J. Schumacher

559 C. Fricker and M.R. Jaïbi
Monotonicity and stability of periodic polling models
Communicated by Prof.dr.ir. O.J. Boxma

560 Ton Geerts
Free end-point linear-quadratic control subject to implicit continuous-time systems: necessary and sufficient conditions for solvability
Communicated by Prof.dr. J. Schumacher

561 Paul G.H. Mulder and Anton L. Hempenius
Expected Utility of Life Time in the Presence of a Chronic Noncommunicable Disease State
Communicated by Prof.dr. B.B. van der Genugten

562 Jan van der Leeuw
The covariance matrix of ARMA-errors in closed form
Communicated by Dr. H.H. Tigelaar

563 J.P.C. Blanc and R.D. van der Mei
Optimization of polling systems with Bernoulli schedules
Communicated by Prof.dr.ir. O.J. Boxma

564 B.B. van der Genugten
Density of the least squares estimator in the multivariate linear model with arbitrarily normal variables
Communicated by Prof.dr. M.H.C. Paardekooper

565 René van den Brink, Robert P. Gilles
Measuring Domination in Directed Graphs
Communicated by Prof.dr. P.H.M. Ruys

566 Harry G. Barkema
The significance of work incentives from bonuses: some new evidence
Communicated by Dr. Th.E. Nijman